BRANDYWINE School District

Embrace, Inspire and Challenge Every Student Every Day

Brandywine High School

1400 Foulk Road • Wilmington, Delaware 19803
(302) 479-1600 • FAX (302) 479-1604

MARK A. HOLODICK, Ed.D.
Superintendent

TO: Students registered for Calculus AB

RE: Summer Assignment

1. Your summer assignment is to cover review sections from Pre-Calculus.

2. Please read the section before you attempt the problems.

3. Your assignment for each section is written on the back of this paper.

4. This summer assignment will be collected on the first day of school. It is worth 30 points.

5. Please show as much work as possible. Just a list of answers will not be acceptable. If the directions say "Graph using the calculator", then always include a sketch of the graph.

6. You will be given the solutions on the second day of school and then you will be able to correct your mistakes.

7. If you do not have a graphing calculator of your own to use, then please stop by Room 119 and I will assign you one for the summer.

Mrs. BL Kidder

Mrs. Brenda L. Kidder

Mathematics Dept Chair

The Brandywine School District is an equal opportunity employer and does not discriminate or deny services on the basis of race, national origin, religion, sex, disability, age or sexual orientation.

<u>SUMMER ASSIGNMENT</u>

☺ **DO NOT WRITE IN THE BOOKLET !!** ☺

1). <u>Section P.2:</u> "Linear Models and Rates of Change"

Pg. 16 # 5 – 11 odd, 23, 25, 26, 28, 30, 32, 37, 39, 40, 55, 59, 63, 69, 70, 71

2). <u>Section P.3</u>: " Functions and Their Graphs"

Pg. 27 # 1, 2, 5, 7, 13, 14, 16 – 24 all, 27, 28, 29 – 35 odd, 39 – 44 all, 47 – 54 all, 57, 59, 61, 65 – 70 all, 74, 93

3). <u>Section P.5</u>: "Solving Equations & Inequalities Algebraically and Graphically"

pg. 31 # 1 - 9 all

*** <u>REMINDER</u>: Do Not Write in Booklet!**

4). <u>Section P.6</u>: "Inverses"

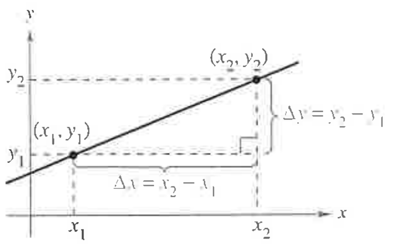
pg. 32 # 1 - 7 all

5). <u>Section P.7</u>: "Trig Review"

pg. 33 # 1 - 19 all

Section P.2 Linear Models and Rates of Change

- Find the slope of a line passing through two points.
- Write the equation of a line with a given point and slope.
- Interpret slope as a ratio or as a rate in a real-life application.
- Sketch the graph of a linear equation in slope-intercept form.
- Write equations of lines that are parallel or perpendicular to a given line.



$\Delta y = y_2 - y_1 =$ change in y
$\Delta x = x_2 - x_1 =$ change in x
Figure P.12

The Slope of a Line

The **slope** of a nonvertical line is a measure of the number of units the line rises (or falls) vertically for each unit of horizontal change from left to right. Consider the two points (x_1, y_1) and (x_2, y_2) on the line in Figure P.12. As you move from left to right along this line, a vertical change of

$$\Delta y = y_2 - y_1 \qquad \text{Change in } y$$

units corresponds to a horizontal change of

$$\Delta x = x_2 - x_1 \qquad \text{Change in } x$$

units. (Δ is the Greek uppercase letter *delta*, and the symbols Δy and Δx are read "delta y" and "delta x.")

Definition of the Slope of a Line

The **slope** m of the nonvertical line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}, \qquad x_1 \neq x_2.$$

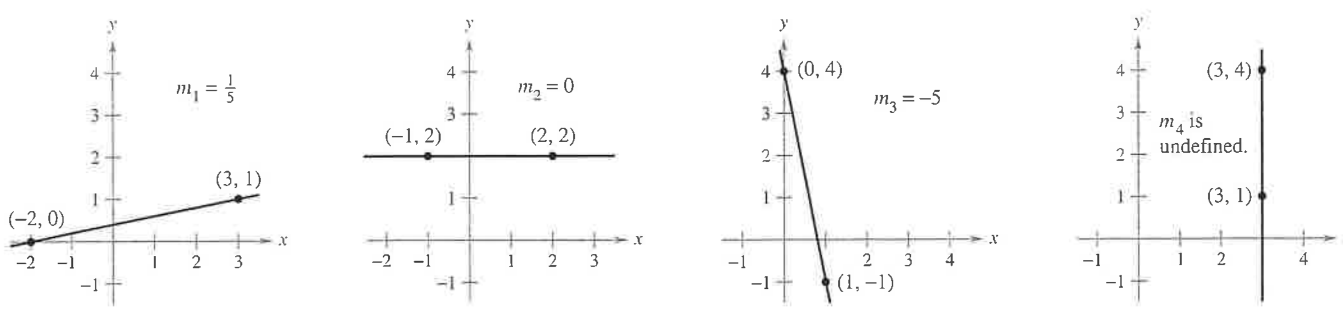
Slope is not defined for vertical lines.

NOTE When using the formula for slope, note that

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-(y_1 - y_2)}{-(x_1 - x_2)} = \frac{y_1 - y_2}{x_1 - x_2}.$$

So, it does not matter in which order you subtract *as long as* you are consistent and both "subtracted coordinates" come from the same point.

Figure P.13 shows four lines: one has a positive slope, one has a slope of zero, one has a negative slope, and one has an "undefined" slope. In general, the greater the absolute value of the slope of a line, the steeper the line is. For instance, in Figure P.13, the line with a slope of -5 is steeper than the line with a slope of $\frac{1}{5}$.



If m is positive, then the line rises from left to right.
Figure P.13

If m is zero, then the line is horizontal.

If m is negative, then the line falls from left to right.

If m is undefined, then the line is vertical.

EXPLORATION

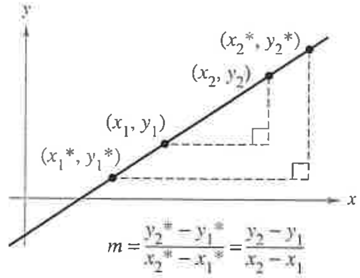
Investigating Equations of Lines Use a graphing utility to graph each of the linear equations. Which point is common to all seven lines? Which value in the equation determines the slope of each line?

a. $y - 4 = -2(x + 1)$
b. $y - 4 = -1(x + 1)$
c. $y - 4 = -\frac{1}{2}(x + 1)$
d. $y - 4 = 0(x + 1)$
e. $y - 4 = \frac{1}{2}(x + 1)$
f. $y - 4 = 1(x + 1)$
g. $y - 4 = 2(x + 1)$

Use your results to write an equation of a line passing through $(-1, 4)$ with a slope of m.

Equations of Lines

Any two points on a nonvertical line can be used to calculate its slope. This can be verified from the similar triangles shown in Figure P.14. (Recall that the ratios of corresponding sides of similar triangles are equal.)



Any two points on a nonvertical line can be used to determine its slope.
Figure P.14

You can write an equation of a nonvertical line if you know the slope of the line and the coordinates of one point on the line. Suppose the slope is m and the point is (x_1, y_1). If (x, y) is any other point on the line, then

$$\frac{y - y_1}{x - x_1} = m.$$

This equation, involving the two variables x and y, can be rewritten in the form $y - y_1 = m(x - x_1)$, which is called the **point-slope equation of a line.**

Point-Slope Equation of a Line

An equation of the line with slope m passing through the point (x_1, y_1) is given by

$$y - y_1 = m(x - x_1).$$

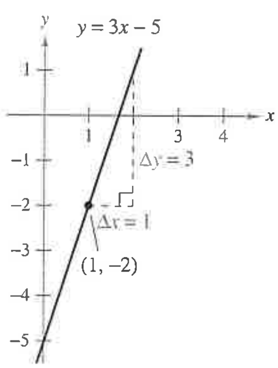
EXAMPLE 1 Finding an Equation of a Line

Find an equation of the line that has a slope of 3 and passes through the point $(1, -2)$.

Solution

$y - y_1 = m(x - x_1)$ Point-slope form
$y - (-2) = 3(x - 1)$ Substitute -2 for y_1, 1 for x_1, and 3 for m.
$y + 2 = 3x - 3$ Simplify.
$y = 3x - 5$ Solve for y.

(See Figure P.15.)

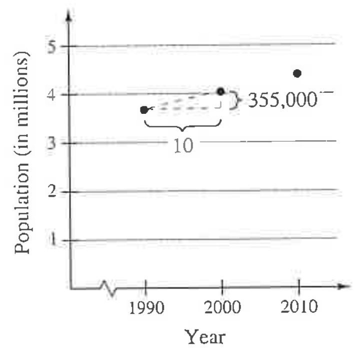


The line with a slope of 3 passing through the point $(1, -2)$
Figure P.15

NOTE Remember that only nonvertical lines have a slope. Consequently, vertical lines cannot be written in point-slope form. For instance, the equation of the vertical line passing through the point $(1, -2)$ is $x = 1$.

Ratios and Rates of Change

The slope of a line can be interpreted as either a *ratio* or a *rate*. If the x- and y-axes have the same unit of measure, the slope has no units and is a **ratio.** If the x- and y-axes have different units of measure, the slope is a rate or **rate of change.** In your study of calculus, you will encounter applications involving both interpretations of slope.



Population of Kentucky in census years
Figure P.16

EXAMPLE 2 Population Growth and Engineering Design

a. The population of Kentucky was 3,687,000 in 1990 and 4,042,000 in 2000. Over this 10-year period, the average rate of change of the population was

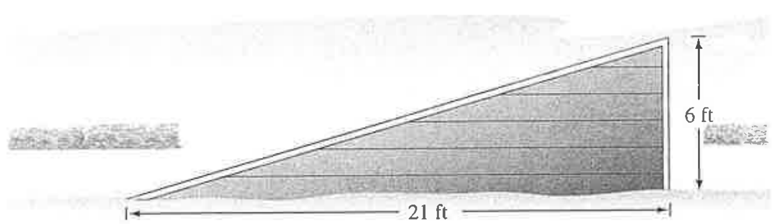
$$\begin{aligned}\text{Rate of change} &= \frac{\text{change in population}}{\text{change in years}}\\ &= \frac{4{,}042{,}000 - 3{,}687{,}000}{2000 - 1990}\\ &= 35{,}500 \text{ people per year.}\end{aligned}$$

If Kentucky's population continues to increase at this same rate for the next 10 years, it will have a 2010 population of 4,397,000 (see Figure P.16). *(Source: U.S. Census Bureau)*

b. In tournament water-ski jumping, the ramp rises to a height of 6 feet on a raft that is 21 feet long, as shown in Figure P.17. The slope of the ski ramp is the ratio of its height (the rise) to the length of its base (the run).

$$\begin{aligned}\text{Slope of ramp} &= \frac{\text{rise}}{\text{run}} \qquad \text{Rise is vertical change, run is horizontal change.}\\ &= \frac{6 \text{ feet}}{21 \text{ feet}}\\ &= \frac{2}{7}\end{aligned}$$

In this case, note that the slope is a ratio and has no units.



Dimensions of a water-ski ramp
Figure P.17

The rate of change found in Example 2(a) is an **average rate of change.** An average rate of change is always calculated over an interval. In this case, the interval is [1990, 2000]. In Chapter 2 you will study another type of rate of change called an *instantaneous rate of change*.

Graphing Linear Models

Many problems in analytic geometry can be classified in two basic categories: (1) Given a graph, what is its equation? and (2) Given an equation, what is its graph? The point-slope equation of a line can be used to solve problems in the first category. However, this form is not especially useful for solving problems in the second category. The form that is better suited to sketching the graph of a line is the **slope-intercept** form of the equation of a line.

The Slope-Intercept Equation of a Line

The graph of the linear equation

$$y = mx + b$$

is a line having a *slope* of m and a *y-intercept* at $(0, b)$.

EXAMPLE 3 Sketching Lines in the Plane

Sketch the graph of each equation.

a. $y = 2x + 1$ **b.** $y = 2$ **c.** $3y + x - 6 = 0$

Solution

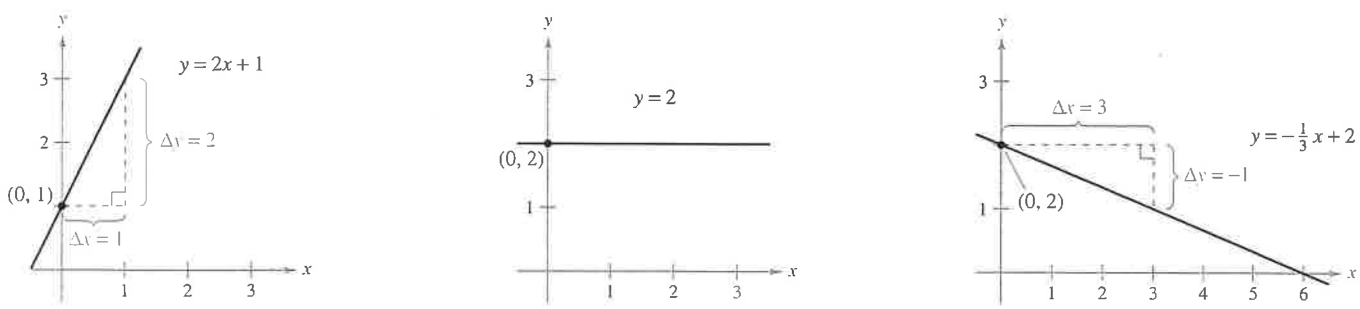
a. Because $b = 1$, the y-intercept is $(0, 1)$. Because the slope is $m = 2$, you know that the line rises two units for each unit it moves to the right, as shown in Figure P.18(a).

b. Because $b = 2$, the y-intercept is $(0, 2)$. Because the slope is $m = 0$, you know that the line is horizontal, as shown in Figure P.18(b).

c. Begin by writing the equation in slope-intercept form.

$$3y + x - 6 = 0 \qquad \text{Write original equation.}$$
$$3y = -x + 6 \qquad \text{Isolate } y\text{-term on the left.}$$
$$y = -\frac{1}{3}x + 2 \qquad \text{Slope-intercept form}$$

In this form, you can see that the y-intercept is $(0, 2)$ and the slope is $m = -\frac{1}{3}$. This means that the line falls one unit for every three units it moves to the right, as shown in Figure P.18(c).



(a) $m = 2$; line rises (b) $m = 0$; line is horizontal (c) $m = -\frac{1}{3}$; line falls

Figure P.18

Because the slope of a vertical line is not defined, its equation cannot be written in the slope-intercept form. However, the equation of any line can be written in the **general form**

$$Ax + By + C = 0 \qquad \text{General form of the equation of a line}$$

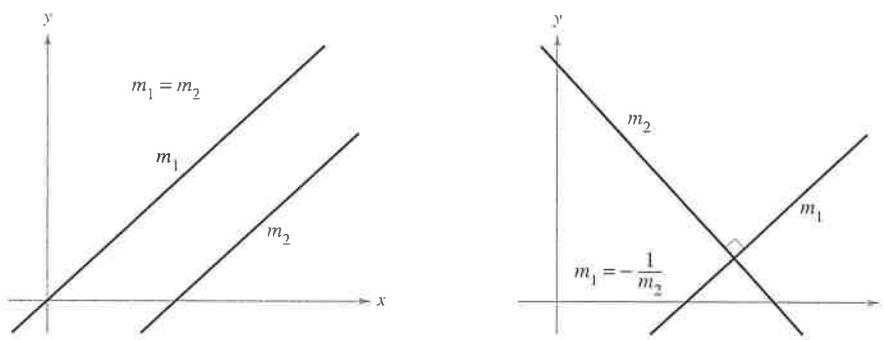
where A and B are not *both* zero. For instance, the vertical line given by $x = a$ can be represented by the general form $x - a = 0$.

Summary of Equations of Lines

1. General form: $Ax + By + C = 0, \quad (A, B \neq 0)$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Point-slope form: $y - y_1 = m(x - x_1)$
5. Slope-intercept form: $y = mx + b$

Parallel and Perpendicular Lines

The slope of a line is a convenient tool for determining whether two lines are parallel or perpendicular, as shown in Figure P.19. Specifically, nonvertical lines with the same slope are parallel and nonvertical lines whose slopes are negative reciprocals are perpendicular.



Parallel lines
Perpendicular lines
Figure P.19

STUDY TIP In mathematics, the phrase "if and only if" is a way of stating two implications in one statement. For instance, the first statement at the right could be rewritten as the following two implications.

a. If two distinct nonvertical lines are parallel, then their slopes are equal.

b. If two distinct nonvertical lines have equal slopes, then they are parallel.

Parallel and Perpendicular Lines

1. Two distinct nonvertical lines are **parallel** if and only if their slopes are equal—that is, if and only if $m_1 = m_2$.
2. Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other—that is, if and only if

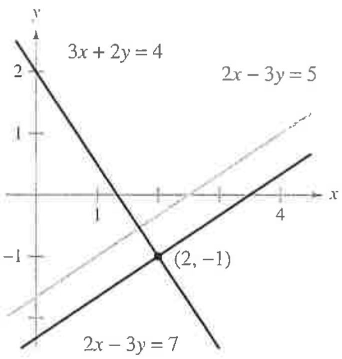
$$m_1 = -\frac{1}{m_2}.$$

EXAMPLE 4 Finding Parallel and Perpendicular Lines

Find the general forms of the equations of the lines that pass through the point $(2, -1)$ and are

a. parallel to the line $2x - 3y = 5$ **b.** perpendicular to the line $2x - 3y = 5$.

(See Figure P.20.)



Lines parallel and perpendicular to $2x - 3y = 5$
Figure P.20

Solution By writing the linear equation $2x - 3y = 5$ in slope-intercept form, $y = \frac{2}{3}x - \frac{5}{3}$, you can see that the given line has a slope of $m = \frac{2}{3}$.

a. The line through $(2, -1)$ that is parallel to the given line also has a slope of $\frac{2}{3}$.

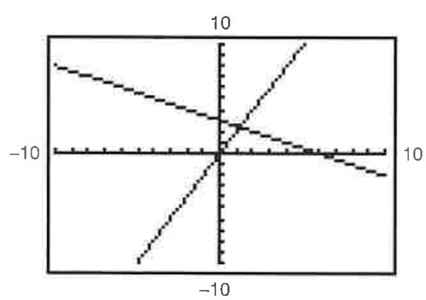
$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-1) = \frac{2}{3}(x - 2)$	Substitute.
$3(y + 1) = 2(x - 2)$	Simplify.
$2x - 3y - 7 = 0$	General form

Note the similarity to the original equation.

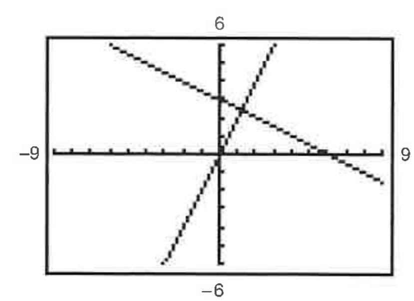
b. Using the negative reciprocal of the slope of the given line, you can determine that the slope of a line perpendicular to the given line is $-\frac{3}{2}$. So, the line through the point $(2, -1)$ that is perpendicular to the given line has the following equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - (-1) = -\frac{3}{2}(x - 2)$	Substitute.
$2(y + 1) = -3(x - 2)$	Simplify.
$3x + 2y - 4 = 0$	General form

TECHNOLOGY PITFALL The slope of a line will appear distorted if you use different tick-mark spacing on the x- and y-axes. For instance, the graphing calculator screens in Figures P.21(a) and P.21(b) both show the lines given by $y = 2x$ and $y = -\frac{1}{2}x + 3$. Because these lines have slopes that are negative reciprocals, they must be perpendicular. In Figure P.21(a), however, the lines don't appear to be perpendicular because the tick-mark spacing on the x-axis is not the same as that on the y-axis. In Figure P.21(b), the lines appear perpendicular because the tick-mark spacing on the x-axis is the same as on the y-axis. This type of viewing window is said to have a *square setting*.



(a) Tick-mark spacing on the x-axis is not the same as tick-mark spacing on the y-axis.



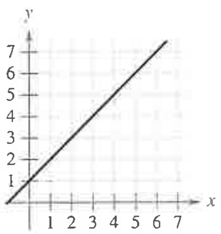
(b) Tick-mark spacing on the x-axis is the same as tick-mark spacing on the y-axis.

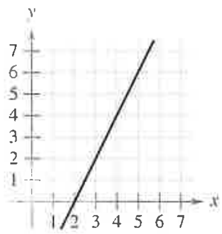
Figure P.21

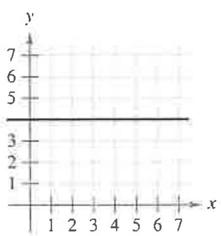
Exercises for Section P.2

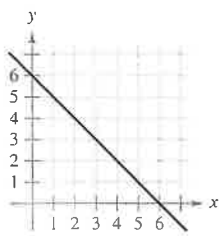
See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

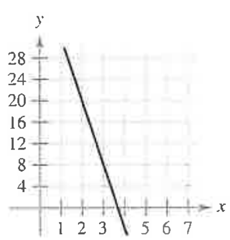
In Exercises 1–6, estimate the slope of the line from its graph. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

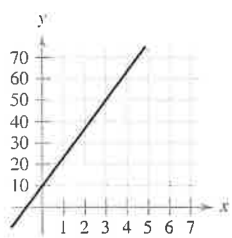
1.


2.


3.


4.


5.


6.


In Exercises 7 and 8, sketch the lines through the point with the indicated slopes. Make the sketches on the same set of coordinate axes.

	Point	*Slopes*			
7.	$(2, 3)$	(a) 1	(b) -2	(c) $-\frac{3}{2}$	(d) Undefined
8.	$(-4, 1)$	(a) 3	(b) -3	(c) $\frac{1}{3}$	(d) 0

In Exercises 9–14, plot the pair of points and find the slope of the line passing through them.

9. $(3, -4), (5, 2)$
10. $(1, 2), (-2, 4)$
11. $(2, 1), (2, 5)$
12. $(3, -2), (4, -2)$
13. $\left(-\frac{1}{2}, \frac{2}{3}\right), \left(-\frac{3}{4}, \frac{1}{6}\right)$
14. $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$

In Exercises 15–18, use the point on the line and the slope of the line to find three additional points that the line passes through. (There is more than one correct answer.)

	Point	*Slope*		*Point*	*Slope*
15.	$(2, 1)$	$m = 0$	16.	$(-3, 4)$	m undefined
17.	$(1, 7)$	$m = -3$	18.	$(-2, -2)$	$m = 2$

19. *Conveyor Design* A moving conveyor is built to rise 1 meter for each 3 meters of horizontal change.
(a) Find the slope of the conveyor.
(b) Suppose the conveyor runs between two floors in a factory. Find the length of the conveyor if the vertical distance between floors is 10 feet.

20. *Rate of Change* Each of the following is the slope of a line representing daily revenue y in terms of time x in days. Use the slope to interpret any change in daily revenue for a one-day increase in time.
(a) $m = 400$ (b) $m = 100$ (c) $m = 0$

21. *Modeling Data* The table shows the populations y (in millions) of the United States for 1996–2001. The variable t represents the time in years, with $t = 6$ corresponding to 1996. *(Source: U.S. Bureau of the Census)*

t	6	7	8	9	10	11
y	269.7	272.9	276.1	279.3	282.3	285.0

(a) Plot the data by hand and connect adjacent points with a line segment.
(b) Use the slope of each line segment to determine the year when the population increased least rapidly.

22. *Modeling Data* The table shows the rate r (in miles per hour) that a vehicle is traveling after t seconds.

t	5	10	15	20	25	30
r	57	74	85	84	61	43

(a) Plot the data by hand and connect adjacent points with a line segment.
(b) Use the slope of each line segment to determine the interval when the vehicle's rate changed most rapidly. How did the rate change?

In Exercises 23–26, find the slope and the y-intercept (if possible) of the line.

23. $x + 5y = 20$
24. $6x - 5y = 15$
25. $x = 4$
26. $y = -1$

In Exercises 27–32, find an equation of the line that passes through the point and has the indicated slope. Sketch the line.

	Point	*Slope*		*Point*	*Slope*
27.	$(0, 3)$	$m = \frac{3}{4}$	28.	$(-1, 2)$	m undefined
29.	$(0, 0)$	$m = \frac{2}{3}$	30.	$(0, 4)$	$m = 0$
31.	$(3, -2)$	$m = 3$	32.	$(-2, 4)$	$m = -\frac{3}{5}$

In Exercises 33–42, find an equation of the line that passes through the points, and sketch the line.

33. $(0, 0), (2, 6)$
34. $(0, 0), (-1, 3)$
35. $(2, 1), (0, -3)$
36. $(-3, -4), (1, 4)$
37. $(2, 8), (5, 0)$
38. $(-3, 6), (1, 2)$
39. $(5, 1), (5, 8)$
40. $(1, -2), (3, -2)$
41. $\left(\frac{1}{2}, \frac{7}{2}\right), \left(0, \frac{3}{4}\right)$
42. $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$

43. Find an equation of the vertical line with x-intercept at 3.

44. Show that the line with intercepts $(a, 0)$ and $(0, b)$ has the following equation.

$$\frac{x}{a} + \frac{y}{b} = 1, \qquad a \neq 0, b \neq 0$$

In Exercises 45–48, use the result of Exercise 44 to write an equation of the line.

45. x-intercept: $(2, 0)$
y-intercept: $(0, 3)$

46. x-intercept: $\left(-\frac{2}{3}, 0\right)$
y-intercept: $(0, -2)$

47. Point on line: $(1, 2)$
x-intercept: $(a, 0)$
y-intercept: $(0, a)$
$(a \neq 0)$

48. Point on line: $(-3, 4)$
x-intercept: $(a, 0)$
y-intercept: $(0, a)$
$(a \neq 0)$

In Exercises 49–56, sketch a graph of the equation.

49. $y = -3$
50. $x = 4$
51. $y = -2x + 1$
52. $y = \frac{1}{3}x - 1$
53. $y - 2 = \frac{3}{2}(x - 1)$
54. $y - 1 = 3(x + 4)$
55. $2x - y - 3 = 0$
56. $x + 2y + 6 = 0$

Square Setting **In Exercises 57 and 58, use a graphing utility to graph both lines in each viewing window. Compare the graphs. Do the lines appear perpendicular? Are the lines perpendicular? Explain.**

57. $y = x + 6,\ y = -x + 2$

(a)
Xmin = -10
Xmax = 10
Xscl = 1
Ymin = -10
Ymax = 10
Yscl = 1

(b)
Xmin = -15
Xmax = 15
Xscl = 1
Ymin = -10
Ymax = 10
Yscl = 1

58. $y = 2x - 3,\ y = -\frac{1}{2}x + 1$

(a)
Xmin = -5
Xmax = 5
Xscl = 1
Ymin = -5
Ymax = 5
Yscl = 1

(b)
Xmin = -6
Xmax = 6
Xscl = 1
Ymin = -4
Ymax = 4
Yscl = 1

In Exercises 59–64, write an equation of the line through the point (a) parallel to the given line and (b) perpendicular to the given line.

	Point	*Line*		*Point*	*Line*
59.	$(2, 1)$	$4x - 2y = 3$	60.	$(-3, 2)$	$x + y = 7$
61.	$\left(\frac{3}{4}, \frac{7}{8}\right)$	$5x - 3y = 0$	62.	$(-6, 4)$	$3x + 4y = 7$
63.	$(2, 5)$	$x = 4$	64.	$(-1, 0)$	$y = -3$

Rate of Change **In Exercises 65–68, you are given the dollar value of a product in 2004 *and* the rate at which the value of the product is expected to change during the next 5 years. Write a linear equation that gives the dollar value V of the product in terms of the year t. (Let $t = 0$ represent 2000.)**

	2004 Value	*Rate*
65.	\$2540	\$125 increase per year
66.	\$156	\$4.50 increase per year
67.	\$20,400	\$2000 decrease per year
68.	\$245,000	\$5600 decrease per year

In Exercises 69 and 70, use a graphing utility to graph the parabolas and find their points of intersection. Find an equation of the line through the points of intersection and graph the line in the same viewing window.

69. $y = x^2$
$y = 4x - x^2$

70. $y = x^2 - 4x + 3$
$y = -x^2 + 2x + 3$

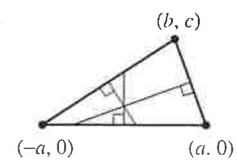
In Exercises 71 and 72, determine whether the points are collinear. (Three points are *collinear* if they lie on the same line.)

71. $(-2, 1), (-1, 0), (2, -2)$
72. $(0, 4), (7, -6), (-5, 11)$

Writing About Concepts

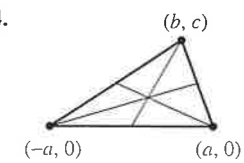
In Exercises 73–75, find the coordinates of the point of intersection of the given segments. Explain your reasoning.

73.



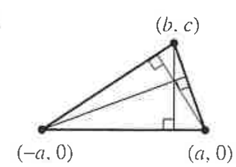
Perpendicular bisectors

74.



Medians

75.



Altitudes

76. Show that the points of intersection in Exercises 73, 74, and 75 are collinear.

77. ***Temperature Conversion*** Find a linear equation that expresses the relationship between the temperature in degrees Celsius C and degrees Fahrenheit F. Use the fact that water freezes at 0°C (32°F) and boils at 100°C (212°F). Use the equation to convert 72°F to degrees Celsius.

78. ***Reimbursed Expenses*** A company reimburses its sales representatives \$150 per day for lodging and meals plus 34¢ per mile driven. Write a linear equation giving the daily cost C to the company in terms of x, the number of miles driven. How much does it cost the company if a sales representative drives 137 miles on a given day?

79. ***Career Choice*** An employee has two options for positions in a large corporation. One position pays \$12.50 per hour *plus* an additional unit rate of \$0.75 per unit produced. The other pays \$9.20 per hour *plus* a unit rate of \$1.30.

(a) Find linear equations for the hourly wages W in terms of x, the number of units produced per hour, for each option.

(b) Use a graphing utility to graph the linear equations and find the point of intersection.

(c) Interpret the meaning of the point of intersection of the graphs in part (b). How would you use this information to select the correct option if the goal were to obtain the highest hourly wage?

80. ***Straight-Line Depreciation*** A small business purchases a piece of equipment for \$875. After 5 years the equipment will be outdated, having no value.

(a) Write a linear equation giving the value y of the equipment in terms of the time x, $0 \le x \le 5$.

(b) Find the value of the equipment when $x = 2$.

(c) Estimate (to two-decimal-place accuracy) the time when the value of the equipment is \$200.

81. ***Apartment Rental*** A real estate office handles an apartment complex with 50 units. When the rent is \$580 per month, all 50 units are occupied. However, when the rent is \$625, the average number of occupied units drops to 47. Assume that the relationship between the monthly rent p and the demand x is linear. (*Note:* The term *demand* refers to the number of occupied units.)

(a) Write a linear equation giving the demand x in terms of the rent p.

(b) *Linear extrapolation* Use a graphing utility to graph the demand equation and use the *trace* feature to predict the number of units occupied if the rent is raised to \$655.

(c) *Linear interpolation* Predict the number of units occupied if the rent is lowered to \$595. Verify graphically.

82. ***Modeling Data*** An instructor gives regular 20-point quizzes and 100-point exams in a mathematics course. Average scores for six students, given as ordered pairs (x, y) where x is the average quiz score and y is the average test score, are (18, 87), (10, 55), (19, 96), (16, 79), (13, 76), and (15, 82).

(a) Use the regression capabilities of a graphing utility to find the least squares regression line for the data.

(b) Use a graphing utility to plot the points and graph the regression line in the same viewing window.

(c) Use the regression line to predict the average exam score for a student with an average quiz score of 17.

(d) Interpret the meaning of the slope of the regression line.

(e) The instructor adds 4 points to the average test score of everyone in the class. Describe the changes in the positions of the plotted points and the change in the equation of the line.

83. ***Tangent Line*** Find an equation of the line tangent to the circle $x^2 + y^2 = 169$ at the point (5, 12).

84. ***Tangent Line*** Find an equation of the line tangent to the circle $(x - 1)^2 + (y - 1)^2 = 25$ at the point $(4, -3)$.

Distance **In Exercises 85–90, find the distance between the point and line, or between the lines, using the formula for the distance between the point (x_1, y_1) and the line $Ax + By + C = 0$.**

$$\textbf{Distance} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

85. Point: (0, 0)
Line: $4x + 3y = 10$

86. Point: (2, 3)
Line: $4x + 3y = 10$

87. Point: $(-2, 1)$
Line: $x - y - 2 = 0$

88. Point: (6, 2)
Line: $x = -1$

89. Line: $x + y = 1$
Line: $x + y = 5$

90. Line: $3x - 4y = 1$
Line: $3x - 4y = 10$

91. Show that the distance between the point (x_1, y_1) and the line $Ax + By + C = 0$ is

$$\text{Distance} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

92. Write the distance d between the point (3, 1) and the line $y = mx + 4$ in terms of m. Use a graphing utility to graph the equation. When is the distance 0? Explain the result geometrically.

93. Prove that the diagonals of a rhombus intersect at right angles. (A rhombus is a quadrilateral with sides of equal lengths.)

94. Prove that the figure formed by connecting consecutive midpoints of the sides of any quadrilateral is a parallelogram.

95. Prove that if the points (x_1, y_1) and (x_2, y_2) lie on the same line as (x_1^*, y_1^*) and (x_2^*, y_2^*), then

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Assume $x_1 \neq x_2$ and $x_1^* \neq x_2^*$.

96. Prove that if the slopes of two nonvertical lines are negative reciprocals of each other, then the lines are perpendicular.

True or False? **In Exercises 97 and 98, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

97. The lines represented by $ax + by = c_1$ and $bx - ay = c_2$ are perpendicular. Assume $a \neq 0$ and $b \neq 0$.

98. It is possible for two lines with positive slopes to be perpendicular to each other.

Section P.3 Functions and Their Graphs

- **Use function notation to represent and evaluate a function.**
- **Find the domain and range of a function.**
- **Sketch the graph of a function.**
- **Identify different types of transformations of functions.**
- **Classify functions and recognize combinations of functions.**

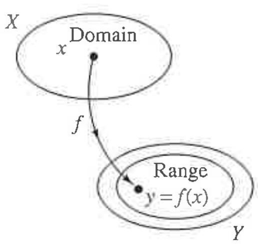
Functions and Function Notation

A **relation** between two sets X and Y is a set of ordered pairs, each of the form (x, y), where x is a member of X and y is a member of Y. A **function** from X to Y is a relation between X and Y that has the property that any two ordered pairs with the same x-value also have the same y-value. The variable x is the **independent variable,** and the variable y is the **dependent variable.**

Many real-life situations can be modeled by functions. For instance, the area A of a circle is a function of the circle's radius r.

$$A = \pi r^2 \qquad A \text{ is a function of } r.$$

In this case r is the independent variable and A is the dependent variable.



A real-valued function f of a real variable
Figure P.22

> **Definition of a Real-Valued Function of a Real Variable**
>
> Let X and Y be sets of real numbers. A **real-valued function f of a real variable x** from X to Y is a correspondence that assigns to each number x in X exactly one number y in Y.
>
> The **domain** of f is the set X. The number y is the **image** of x under f and is denoted by $f(x)$, which is called the **value of f at x.** The **range** of f is a subset of Y and consists of all images of numbers in X (see Figure P.22).

Functions can be specified in a variety of ways. In this text, however, we will concentrate primarily on functions that are given by equations involving the dependent and independent variables. For instance, the equation

$$x^2 + 2y = 1 \qquad \text{Equation in implicit form}$$

defines y, the dependent variable, as a function of x, the independent variable. To **evaluate** this function (that is, to find the y-value that corresponds to a given x-value), it is convenient to isolate y on the left side of the equation.

$$y = \frac{1}{2}(1 - x^2) \qquad \text{Equation in explicit form}$$

Using f as the name of the function, you can write this equation as

$$f(x) = \frac{1}{2}(1 - x^2). \qquad \text{Function notation}$$

The original equation, $x^2 + 2y = 1$, **implicitly** defines y as a function of x. When you solve the equation for y, you are writing the equation in **explicit** form.

Function notation has the advantage of clearly identifying the dependent variable as $f(x)$ while at the same time telling you that x is the independent variable and that the function itself is "f." The symbol $f(x)$ is read "f of x." Function notation allows you to be less wordy. Instead of asking "What is the value of y that corresponds to $x = 3$?" you can ask "What is $f(3)$?"

FUNCTION NOTATION

The word *function* was first used by Gottfried Wilhelm Leibniz in 1694 as a term to denote any quantity connected with a curve, such as the coordinates of a point on a curve or the slope of a curve. Forty years later, Leonhard Euler used the word "function" to describe any expression made up of a variable and some constants. He introduced the notation $y = f(x)$.

In an equation that defines a function, the role of the variable x is simply that of a placeholder. For instance, the function given by

$$f(x) = 2x^2 - 4x + 1$$

can be described by the form

$$f(\quad) = 2(\quad)^2 - 4(\quad) + 1$$

where parentheses are used instead of x. To evaluate $f(-2)$, simply place -2 in each set of parentheses.

$$\begin{aligned} f(-2) &= 2(-2)^2 - 4(-2) + 1 && \text{Substitute } -2 \text{ for } x. \\ &= 2(4) + 8 + 1 && \text{Simplify.} \\ &= 17 && \text{Simplify.} \end{aligned}$$

NOTE Although f is often used as a convenient function name and x as the independent variable, you can use other symbols. For instance, the following equations all define the same function.

$$\begin{aligned} f(x) &= x^2 - 4x + 7 && \text{Function name is } f\text{, independent variable is } x. \\ f(t) &= t^2 - 4t + 7 && \text{Function name is } f\text{, independent variable is } t. \\ g(s) &= s^2 - 4s + 7 && \text{Function name is } g\text{, independent variable is } s. \end{aligned}$$

EXAMPLE 1 Evaluating a Function

For the function f defined by $f(x) = x^2 + 7$, evaluate each expression.

a. $f(3a)$ **b.** $f(b - 1)$ **c.** $\dfrac{f(x + \Delta x) - f(x)}{\Delta x}, \quad \Delta x \neq 0$

Solution

a. $$\begin{aligned} f(3a) &= (3a)^2 + 7 && \text{Substitute } 3a \text{ for } x. \\ &= 9a^2 + 7 && \text{Simplify.} \end{aligned}$$

b. $$\begin{aligned} f(b - 1) &= (b - 1)^2 + 7 && \text{Substitute } b - 1 \text{ for } x. \\ &= b^2 - 2b + 1 + 7 && \text{Expand binomial.} \\ &= b^2 - 2b + 8 && \text{Simplify.} \end{aligned}$$

c. $$\begin{aligned} \frac{f(x + \Delta x) - f(x)}{\Delta x} &= \frac{[(x + \Delta x)^2 + 7] - (x^2 + 7)}{\Delta x} \\ &= \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 7 - x^2 - 7}{\Delta x} \\ &= \frac{2x\Delta x + (\Delta x)^2}{\Delta x} \\ &= \frac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}} \\ &= 2x + \Delta x, \quad \Delta x \neq 0 \end{aligned}$$

STUDY TIP In calculus, it is important to communicate clearly the domain of a function or expression. For instance, in Example 1(c) the two expressions

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} \quad \text{and} \quad 2x + \Delta x,$$
$$\Delta x \neq 0$$

are equivalent because $\Delta x = 0$ is excluded from the domain of each expression. Without a stated domain restriction, the two expressions would not be equivalent.

NOTE The expression in Example 1(c) is called a *difference quotient* and has a special significance in calculus. You will learn more about this in Chapter 2.

The Domain and Range of a Function

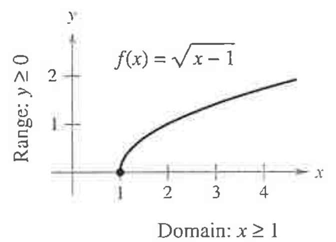
The domain of a function can be described explicitly, or it may be described *implicitly* by an equation used to define the function. The implied domain is the set of all real numbers for which the equation is defined, whereas an explicitly defined domain is one that is given along with the function. For example, the function given by

$$f(x) = \frac{1}{x^2 - 4}, \qquad 4 \le x \le 5$$

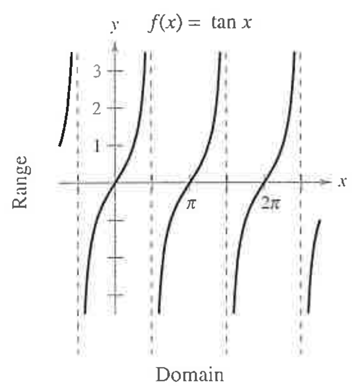
has an explicitly defined domain given by $\{x\colon 4 \le x \le 5\}$. On the other hand, the function given by

$$g(x) = \frac{1}{x^2 - 4}$$

has an implied domain that is the set $\{x\colon x \neq \pm 2\}$.



(a) The domain of f is $[1, \infty)$ and the range is $[0, \infty)$.



(b) The domain of f is all x-values such that $x \neq \frac{\pi}{2} + n\pi$ and the range is $(-\infty, \infty)$.

Figure P.23

EXAMPLE 2 Finding the Domain and Range of a Function

a. The domain of the function

$$f(x) = \sqrt{x - 1}$$

is the set of all x-values for which $x - 1 \ge 0$, which is the interval $[1, \infty)$. To find the range observe that $f(x) = \sqrt{x - 1}$ is never negative. So, the range is the interval $[0, \infty)$, as indicated in Figure P.23(a).

b. The domain of the tangent function, as shown in Figure P.23(b),

$$f(x) = \tan x$$

is the set of all x-values such that

$$x \neq \frac{\pi}{2} + n\pi, \qquad n \text{ is an integer.} \qquad \text{Domain of tangent function}$$

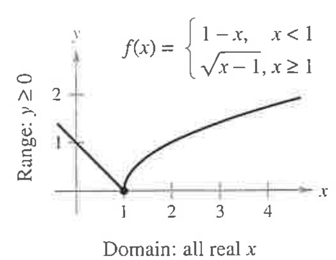
The range of this function is the set of all real numbers. For a review of the characteristics of this and other trigonometric functions, see Appendix D.

EXAMPLE 3 A Function Defined by More than One Equation

Determine the domain and range of the function.

$$f(x) = \begin{cases} 1 - x, & \text{if } x < 1 \\ \sqrt{x - 1}, & \text{if } x \ge 1 \end{cases}$$

Solution Because f is defined for $x < 1$ and $x \ge 1$, the domain is the entire set of real numbers. On the portion of the domain for which $x \ge 1$, the function behaves as in Example 2(a). For $x < 1$, the values of $1 - x$ are positive. So, the range of the function is the interval $[0, \infty)$. (See Figure P.24.)

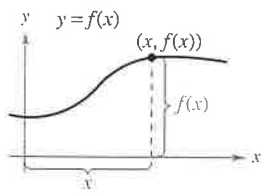


The domain of f is $(-\infty, \infty)$ and the range is $[0, \infty)$.

Figure P.24

A function from X to Y is **one-to-one** if to each y-value in the range there corresponds exactly one x-value in the domain. For instance, the function given in Example 2(a) is one-to-one, whereas the functions given in Examples 2(b) and 3 are not one-to-one. A function from X to Y is **onto** if its range consists of all of Y.

The Graph of a Function

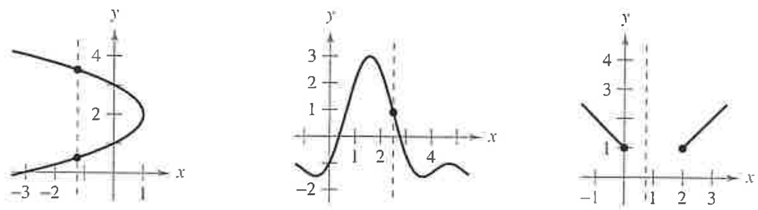


The graph of a function
Figure P.25

The graph of the function $y = f(x)$ consists of all points $(x, f(x))$, where x is in the domain of f. In Figure P.25, note that

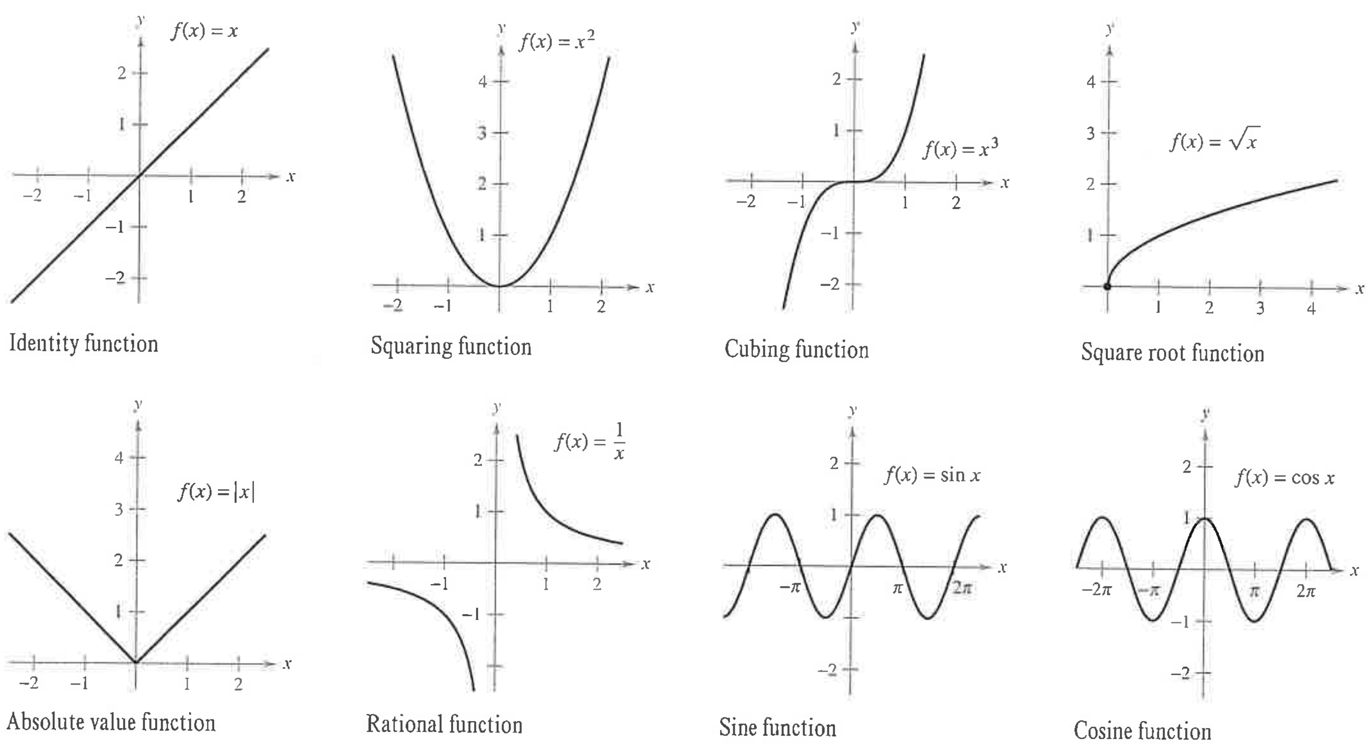
$$x = \text{the directed distance from the } y\text{-axis}$$
$$f(x) = \text{the directed distance from the } x\text{-axis.}$$

A vertical line can intersect the graph of a function of x at most *once*. This observation provides a convenient visual test, called the **Vertical Line Test,** for functions of x. That is, a graph in the coordinate plane is the graph of a function of f if and only if no vertical line intersects the graph at more than one point. For example, in Figure P.26(a), you can see that the graph does not define y as a function of x because a vertical line intersects the graph twice, whereas in Figures P.26(b) and (c), the graphs do define y as a function of x.



(a) Not a function of x (b) A function of x (c) A function of x
Figure P.26

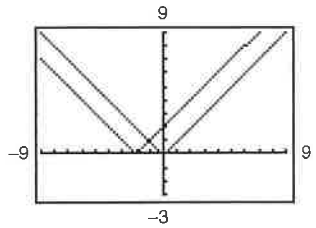
Figure P.27 shows the graphs of eight basic functions. You should be able to recognize these graphs. (Graphs of the other four basic trigonometric functions are shown in Appendix D.)



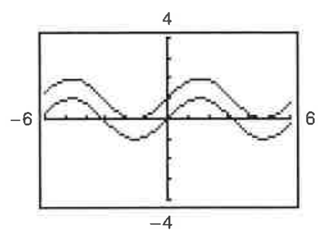
The graphs of eight basic functions
Figure P.27

EXPLORATION

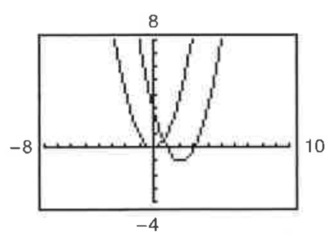
Writing Equations for Functions Each of the graphing utility screens below shows the graph of one of the eight basic functions shown on page 22. Each screen also shows a transformation of the graph. Describe the transformation. Then use your description to write an equation for the transformation.



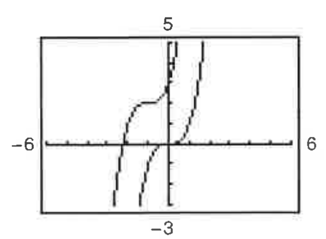
a.



b.



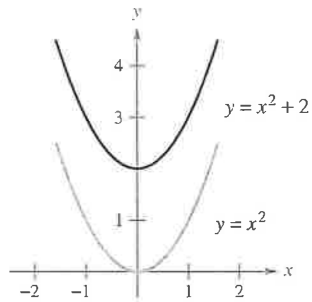
c.



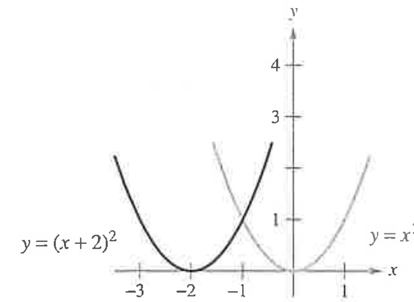
d.

Transformations of Functions

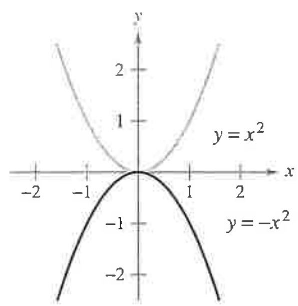
Some families of graphs have the same basic shape. For example, compare the graph of $y = x^2$ with the graphs of the four other quadratic functions shown in Figure P.28.



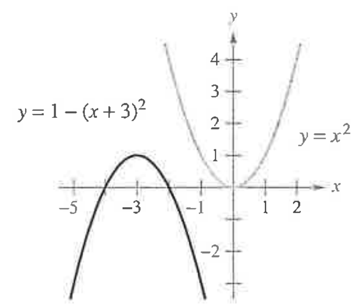
(a) Vertical shift upward



(b) Horizontal shift to the left



(c) Reflection



(d) Shift left, reflect, and shift upward

Figure P.28

Each of the graphs in Figure P.28 is a **transformation** of the graph of $y = x^2$. The three basic types of transformations illustrated by these graphs are vertical shifts, horizontal shifts, and reflections. Function notation lends itself well to describing transformations of graphs in the plane. For instance, if $f(x) = x^2$ is considered to be the original function in Figure P.28, the transformations shown can be represented by the following equations.

$y = f(x) + 2$	Vertical shift up 2 units
$y = f(x + 2)$	Horizontal shift to the left 2 units
$y = -f(x)$	Reflection about the x-axis
$y = -f(x + 3) + 1$	Shift left 3 units, reflect about x-axis, and shift up 1 unit

Basic Types of Transformations $(c > 0)$

Original graph:	$y = f(x)$
Horizontal shift c units to the **right**:	$y = f(x - c)$
Horizontal shift c units to the **left**:	$y = f(x + c)$
Vertical shift c units **downward**:	$y = f(x) - c$
Vertical shift c units **upward**:	$y = f(x) + c$
Reflection (about the x-axis):	$y = -f(x)$
Reflection (about the y-axis):	$y = f(-x)$
Reflection (about the origin):	$y = -f(-x)$

Bettmann/Corbis

LEONHARD EULER (1707–1783)

In addition to making major contributions to almost every branch of mathematics, Euler was one of the first to apply calculus to real-life problems in physics. His extensive published writings include such topics as shipbuilding, acoustics, optics, astronomy, mechanics, and magnetism.

Classifications and Combinations of Functions

The modern notion of a function is derived from the efforts of many seventeenth- and eighteenth-century mathematicians. Of particular note was Leonhard Euler, to whom we are indebted for the function notation $y = f(x)$. By the end of the eighteenth century, mathematicians and scientists had concluded that many real-world phenomena could be represented by mathematical models taken from a collection of functions called **elementary functions.** Elementary functions fall into three categories.

1. Algebraic functions (polynomial, radical, rational)
2. Trigonometric functions (sine, cosine, tangent, and so on)
3. Exponential and logarithmic functions

You can review the trigonometric functions in Appendix D. The other nonalgebraic functions, such as the inverse trigonometric functions and the exponential and logarithmic functions, are introduced in Chapter 5.

The most common type of algebraic function is a **polynomial function**

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_2x^2 + a_1x + a_0, \qquad a_n \neq 0$$

where the positive integer n is the **degree** of the polynomial function. The constants a_i are **coefficients,** with a_n the **leading coefficient** and a_0 the **constant term** of the polynomial function. It is common practice to use subscript notation for coefficients of general polynomial functions, but for polynomial functions of low degree, the following simpler forms are often used.

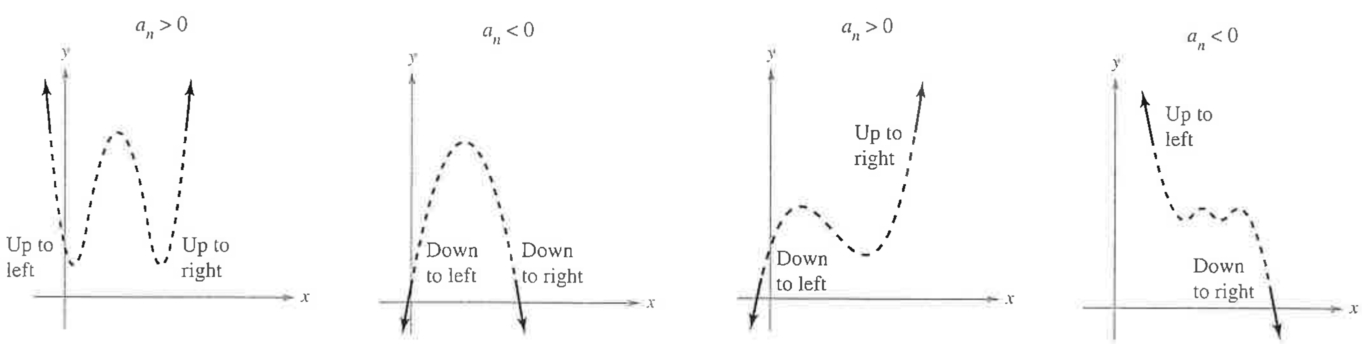
Zeroth degree:	$f(x) = a$	Constant function
First degree:	$f(x) = ax + b$	Linear function
Second degree:	$f(x) = ax^2 + bx + c$	Quadratic function
Third degree:	$f(x) = ax^3 + bx^2 + cx + d$	Cubic function

FOR FURTHER INFORMATION For more on the history of the concept of a function, see the article "Evolution of the Function Concept: A Brief Survey" by Israel Kleiner in *The College Mathematics Journal.* To view this article, go to the website *www.matharticles.com.*

Although the graph of a nonconstant polynomial function can have several turns, eventually the graph will rise or fall without bound as x moves to the right or left. Whether the graph of

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_2x^2 + a_1x + a_0$$

eventually rises or falls can be determined by the function's degree (odd or even) and by the leading coefficient a_n, as indicated in Figure P.29. Note that the dashed portions of the graphs indicate that the **Leading Coefficient Test** determines *only* the right and left behavior of the graph.



Graphs of polynomial functions of even degree

Graphs of polynomial functions of odd degree

The Leading Coefficient Test for polynomial functions
Figure P.29

Just as a rational number can be written as the quotient of two integers, a **rational function** can be written as the quotient of two polynomials. Specifically, a function f is rational if it has the form

$$f(x) = \frac{p(x)}{q(x)}, \quad q(x) \neq 0$$

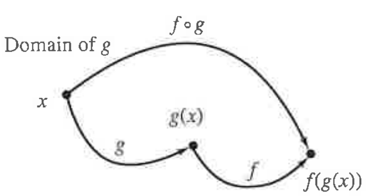
where $p(x)$ and $q(x)$ are polynomials.

Polynomial functions and rational functions are examples of **algebraic functions.** An algebraic function of x is one that can be expressed as a finite number of sums, differences, multiples, quotients, and radicals involving x^n. For example, $f(x) = \sqrt{x+1}$ is algebraic. Functions that are not algebraic are **transcendental.** For instance, the trigonometric functions are transcendental.

Two functions can be combined in various ways to create new functions. For example, given $f(x) = 2x - 3$ and $g(x) = x^2 + 1$, you can form the functions shown.

$$(f+g)(x) = f(x) + g(x) = (2x - 3) + (x^2 + 1) \qquad \text{Sum}$$
$$(f-g)(x) = f(x) - g(x) = (2x - 3) - (x^2 + 1) \qquad \text{Difference}$$
$$(fg)(x) = f(x)g(x) = (2x - 3)(x^2 + 1) \qquad \text{Product}$$
$$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{2x - 3}{x^2 + 1} \qquad \text{Quotient}$$

You can combine two functions in yet another way, called **composition.** The resulting function is called a **composite function.**



The domain of the composite function $f \circ g$
Figure P.30

> **Definition of Composite Function**
>
> Let f and g be functions. The function given by $(f \circ g)(x) = f(g(x))$ is called the **composite** of f with g. The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f (see Figure P.30).

The composite of f with g may not be equal to the composite of g with f.

EXAMPLE 4 Finding Composite Functions

Given $f(x) = 2x - 3$ and $g(x) = \cos x$, find each composite function.

a. $f \circ g$ **b.** $g \circ f$

Solution

a. $(f \circ g)(x) = f(g(x))$ — Definition of $f \circ g$
$= f(\cos x)$ — Substitute $\cos x$ for $g(x)$.
$= 2(\cos x) - 3$ — Definition of $f(x)$
$= 2\cos x - 3$ — Simplify.

b. $(g \circ f)(x) = g(f(x))$ — Definition of $g \circ f$
$= g(2x - 3)$ — Substitute $2x - 3$ for $f(x)$.
$= \cos(2x - 3)$ — Definition of $g(x)$

Note that $(f \circ g)(x) \neq (g \circ f)(x)$.

EXPLORATION

Use a graphing utility to graph each function. Determine whether the function is *even*, *odd*, or *neither*.

$$f(x) = x^2 - x^4$$
$$g(x) = 2x^3 + 1$$
$$h(x) = x^5 - 2x^3 + x$$
$$j(x) = 2 - x^6 - x^8$$
$$k(x) = x^5 - 2x^4 + x - 2$$
$$p(x) = x^9 + 3x^5 - x^3 + x$$

Describe a way to identify a function as odd or even by inspecting the equation.

In Section P.1, an x-intercept of a graph was defined to be a point $(a, 0)$ at which the graph crosses the x-axis. If the graph represents a function f, the number a is a **zero** of f. In other words, *the zeros of a function f are the solutions of the equation $f(x) = 0$.* For example, the function $f(x) = x - 4$ has a zero at $x = 4$ because $f(4) = 0$.

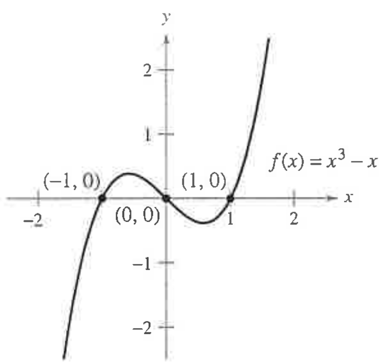
In Section P.1 you also studied different types of symmetry. In the terminology of functions, a function is **even** if its graph is symmetric with respect to the y-axis, and is **odd** if its graph is symmetric with respect to the origin. The symmetry tests in Section P.1 yield the following test for even and odd functions.

Test for Even and Odd Functions

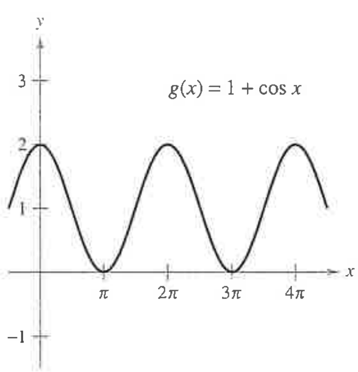
The function $y = f(x)$ is **even** if $f(-x) = f(x)$.

The function $y = f(x)$ is **odd** if $f(-x) = -f(x)$.

NOTE Except for the constant function $f(x) = 0$, the graph of a function of x cannot have symmetry with respect to the x-axis because it then would fail the Vertical Line Test for the graph of the function.



(a) Odd function



(b) Even function

Figure P.31

EXAMPLE 5 Even and Odd Functions and Zeros of Functions

Determine whether each function is even, odd, or neither. Then find the zeros of the function.

a. $f(x) = x^3 - x$ **b.** $g(x) = 1 + \cos x$

Solution

a. This function is odd because

$$f(-x) = (-x)^3 - (-x) = -x^3 + x = -(x^3 - x) = -f(x).$$

The zeros of f are found as shown.

$$x^3 - x = 0 \qquad \text{Let } f(x) = 0.$$
$$x(x^2 - 1) = x(x - 1)(x + 1) = 0 \qquad \text{Factor.}$$
$$x = 0, 1, -1 \qquad \text{Zeros of } f$$

See Figure P.31(a).

b. This function is even because

$$g(-x) = 1 + \cos(-x) = 1 + \cos x = g(x). \qquad \cos(-x) = \cos(x)$$

The zeros of g are found as shown.

$$1 + \cos x = 0 \qquad \text{Let } g(x) = 0.$$
$$\cos x = -1 \qquad \text{Subtract 1 from each side.}$$
$$x = (2n + 1)\pi, \ n \text{ is an integer.} \qquad \text{Zeros of } g$$

See Figure P.31(b).

NOTE Each of the functions in Example 5 is either even or odd. However, some functions, such as $f(x) = x^2 + x + 1$, are neither even nor odd.

Exercises for Section P.3

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1 and 2, use the graphs of f and g to answer the following.

(a) Identify the domains and ranges of f and g.

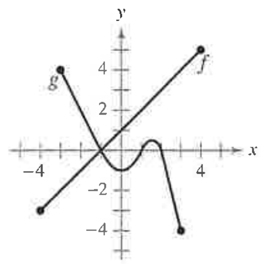
(b) Identify $f(-2)$ and $g(3)$.

(c) For what value(s) of x is $f(x) = g(x)$?

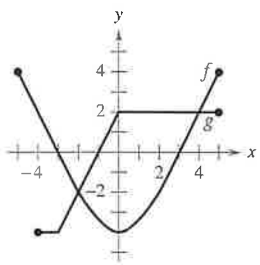
(d) Estimate the solution(s) of $f(x) = 2$.

(e) Estimate the solutions of $g(x) = 0$.

1.



2.



In Exercises 3–12, evaluate (if possible) the function at the given value(s) of the independent variable. Simplify the results.

3. $f(x) = 2x - 3$
(a) $f(0)$
(b) $f(-3)$
(c) $f(b)$
(d) $f(x - 1)$

4. $f(x) = \sqrt{x + 3}$
(a) $f(-2)$
(b) $f(6)$
(c) $f(-5)$
(d) $f(x + \Delta x)$

5. $g(x) = 3 - x^2$
(a) $g(0)$
(b) $g(\sqrt{3})$
(c) $g(-2)$
(d) $g(t - 1)$

6. $g(x) = x^2(x - 4)$
(a) $g(4)$
(b) $g(\frac{3}{2})$
(c) $g(c)$
(d) $g(t + 4)$

7. $f(x) = \cos 2x$
(a) $f(0)$
(b) $f(-\pi/4)$
(c) $f(\pi/3)$

8. $f(x) = \sin x$
(a) $f(\pi)$
(b) $f(5\pi/4)$
(c) $f(2\pi/3)$

9. $f(x) = x^3$

$$\frac{f(x + \Delta x) - f(x)}{\Delta x}$$

10. $f(x) = 3x - 1$

$$\frac{f(x) - f(1)}{x - 1}$$

11. $f(x) = \dfrac{1}{\sqrt{x - 1}}$

$$\frac{f(x) - f(2)}{x - 2}$$

12. $f(x) = x^3 - x$

$$\frac{f(x) - f(1)}{x - 1}$$

In Exercises 13–18, find the domain and range of the function.

13. $h(x) = -\sqrt{x + 3}$

14. $g(x) = x^2 - 5$

15. $f(t) = \sec \dfrac{\pi t}{4}$

16. $h(t) = \cot t$

17. $f(x) = \dfrac{1}{x}$

18. $g(x) = \dfrac{2}{x - 1}$

In Exercises 19-24, find the domain of the function.

19. $f(x) = \sqrt{x} + \sqrt{1 - x}$

20. $f(x) = \sqrt{x^2 - 3x + 2}$

21. $g(x) = \dfrac{2}{1 - \cos x}$

22. $h(x) = \dfrac{1}{\sin x - \frac{1}{2}}$

23. $f(x) = \dfrac{1}{|x + 3|}$

24. $g(x) = \dfrac{1}{|x^2 - 4|}$

In Exercises 25–28, evaluate the function as indicated. Determine its domain and range.

25. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1)$ (b) $f(0)$ (c) $f(2)$ (d) $f(t^2 + 1)$

26. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2)$ (b) $f(0)$ (c) $f(1)$ (d) $f(s^2 + 2)$

27. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$

(a) $f(-3)$ (b) $f(1)$ (c) $f(3)$ (d) $f(b^2 + 1)$

28. $f(x) = \begin{cases} \sqrt{x + 4}, & x \leq 5 \\ (x - 5)^2, & x > 5 \end{cases}$

(a) $f(-3)$ (b) $f(0)$ (c) $f(5)$ (d) $f(10)$

In Exercises 29–36, sketch a graph of the function and find its domain and range. Use a graphing utility to verify your graph.

29. $f(x) = 4 - x$

30. $g(x) = \dfrac{4}{x}$

31. $h(x) = \sqrt{x - 1}$

32. $f(x) = \frac{1}{2}x^3 + 2$

33. $f(x) = \sqrt{9 - x^2}$

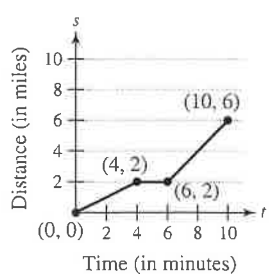
34. $f(x) = x + \sqrt{4 - x^2}$

35. $g(t) = 2 \sin \pi t$

36. $h(\theta) = -5 \cos \dfrac{\theta}{2}$

Writing About Concepts

37. The graph of the distance that a student drives in a 10-minute trip to school is shown in the figure. Give a verbal description of characteristics of the student's drive to school.

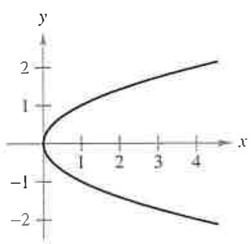


Writing About Concepts (continued)

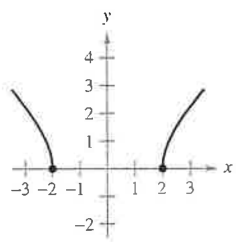
38. A student who commutes 27 miles to attend college remembers, after driving a few minutes, that a term paper that is due has been forgotten. Driving faster than usual, the student returns home, picks up the paper, and once again starts toward school. Sketch a possible graph of the student's distance from home as a function of time.

In Exercises 39–42, use the Vertical Line Test to determine whether y is a function of x. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

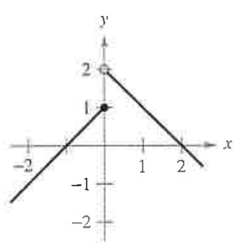
39. $x - y^2 = 0$



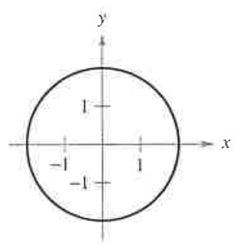
40. $\sqrt{x^2 - 4} - y = 0$



41. $y = \begin{cases} x + 1, & x \le 0 \\ -x + 2, & x > 0 \end{cases}$



42. $x^2 + y^2 = 4$



In Exercises 43–46, determine whether y is a function of x.

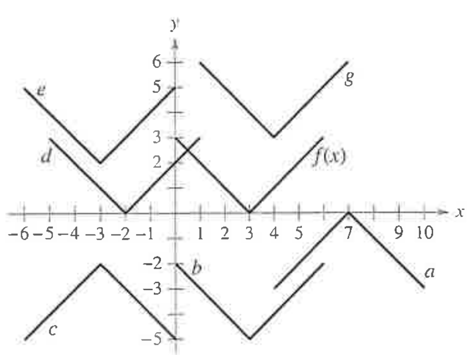
43. $x^2 + y^2 = 4$

44. $x^2 + y = 4$

45. $y^2 = x^2 - 1$

46. $x^2y - x^2 + 4y = 0$

In Exercises 47–52, use the graph of $y = f(x)$ to match the function with its graph.



47. $y = f(x + 5)$

48. $y = f(x) - 5$

49. $y = -f(-x) - 2$

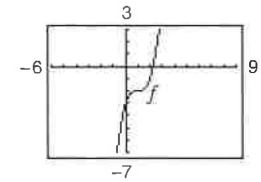
50. $y = -f(x - 4)$

51. $y = f(x + 6) + 2$

52. $y = f(x - 1) + 3$

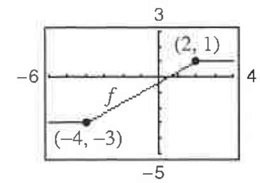
53. Use the graph of f shown in the figure to sketch the graph of each function. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

(a) $f(x + 3)$ (b) $f(x - 1)$
(c) $f(x) + 2$ (d) $f(x) - 4$
(e) $3f(x)$ (f) $\frac{1}{4}f(x)$



54. Use the graph of f shown in the figure to sketch the graph of each function. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

(a) $f(x - 4)$ (b) $f(x + 2)$
(c) $f(x) + 4$ (d) $f(x) - 1$
(e) $2f(x)$ (f) $\frac{1}{2}f(x)$



55. Use the graph of $f(x) = \sqrt{x}$ to sketch the graph of each function. In each case, describe the transformation.

(a) $y = \sqrt{x} + 2$ (b) $y = -\sqrt{x}$ (c) $y = \sqrt{x - 2}$

56. Specify a sequence of transformations that will yield each graph of h from the graph of the function $f(x) = \sin x$.

(a) $h(x) = \sin\left(x + \dfrac{\pi}{2}\right) + 1$ (b) $h(x) = -\sin(x - 1)$

57. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, evaluate each expression.

(a) $f(g(1))$ (b) $g(f(1))$ (c) $g(f(0))$
(d) $f(g(-4))$ (e) $f(g(x))$ (f) $g(f(x))$

58. Given $f(x) = \sin x$ and $g(x) = \pi x$, evaluate each expression.

(a) $f(g(2))$ (b) $f\left(g\left(\dfrac{1}{2}\right)\right)$ (c) $g(f(0))$
(d) $g\left(f\left(\dfrac{\pi}{4}\right)\right)$ (e) $f(g(x))$ (f) $g(f(x))$

In Exercises 59–62, find the composite functions $(f \circ g)$ and $(g \circ f)$. What is the domain of each composite function? Are the two composite functions equal?

59. $f(x) = x^2$
$g(x) = \sqrt{x}$

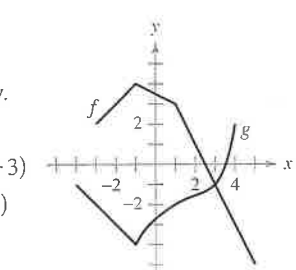
60. $f(x) = x^2 - 1$
$g(x) = \cos x$

61. $f(x) = \dfrac{3}{x}$
$g(x) = x^2 - 1$

62. $f(x) = \dfrac{1}{x}$
$g(x) = \sqrt{x + 2}$

63. Use the graphs of f and g to evaluate each expression. If the result is undefined, explain why.

(a) $(f \circ g)(3)$ (b) $g(f(2))$
(c) $g(f(5))$ (d) $(f \circ g)(-3)$
(e) $(g \circ f)(-1)$ (f) $f(g(-1))$



64. ***Ripples*** A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius (in feet) of the outer ripple is given by $r(t) = 0.6t$, where t is the time in seconds after the pebble strikes the water. The area of the circle is given by the function $A(r) = \pi r^2$. Find and interpret $(A \circ r)(t)$.

Think About It **In Exercises 65 and 66, $F(x) = f \circ g \circ h$. Identify functions for f, g, and h. (There are many correct answers.)**

65. $F(x) = \sqrt{2x - 2}$

66. $F(x) = -4\sin(1 - x)$

In Exercises 67–70, determine whether the function is even, odd, or neither. Use a graphing utility to verify your result.

67. $f(x) = x^2(4 - x^2)$

68. $f(x) = \sqrt[3]{x}$

69. $f(x) = x\cos x$

70. $f(x) = \sin^2 x$

Think About It **In Exercises 71 and 72, find the coordinates of a second point on the graph of a function f if the given point is on the graph and the function is (a) even and (b) odd.**

71. $\left(-\frac{3}{2}, 4\right)$

72. $(4, 9)$

73. The graphs of f, g, and h are shown in the figure. Decide whether each function is even, odd, or neither.

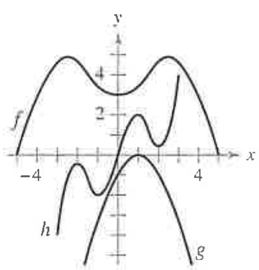


Figure for 73

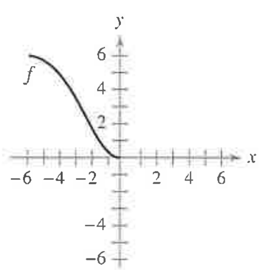


Figure for 74

74. The domain of the function f shown in the figure is $-6 \le x \le 6$.

(a) Complete the graph of f given that f is even.

(b) Complete the graph of f given that f is odd.

Writing Functions **In Exercises 75–78, write an equation for a function that has the given graph.**

75. Line segment connecting $(-4, 3)$ and $(0, -5)$

76. Line segment connecting $(1, 2)$ and $(5, 5)$

77. The bottom half of the parabola $x + y^2 = 0$

78. The bottom half of the circle $x^2 + y^2 = 4$

Modeling Data **In Exercises 79–82, match the data with a function from the following list.**

(i) $f(x) = cx$

(ii) $g(x) = cx^2$

(iii) $h(x) = c\sqrt{|x|}$

(iv) $r(x) = c/x$

Determine the value of the constant c for each function such that the function fits the data shown in the table.

79.

x	-4	-1	0	1	4
y	-32	-2	0	-2	-32

80.

x	-4	-1	0	1	4
y	-1	$-\frac{1}{4}$	0	$\frac{1}{4}$	1

81.

x	-4	-1	0	1	4
y	-8	-32	Undef.	32	8

82.

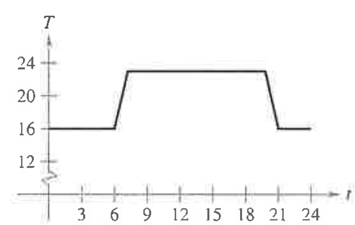
x	-4	-1	0	1	4
y	6	3	0	3	6

83. ***Graphical Reasoning*** An electronically controlled thermostat is programmed to lower the temperature during the night automatically (see figure). The temperature T in degrees Celsius is given in terms of t, the time in hours on a 24-hour clock.

(a) Approximate $T(4)$ and $T(15)$.

(b) The thermostat is reprogrammed to produce a temperature $H(t) = T(t - 1)$. How does this change the temperature? Explain.

(c) The thermostat is reprogrammed to produce a temperature $H(t) = T(t) - 1$. How does this change the temperature? Explain.

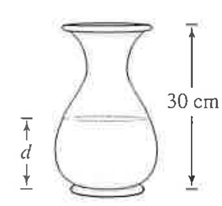


84. Water runs into a vase of height 30 centimeters at a constant rate. The vase is full after 5 seconds. Use this information and the shape of the vase shown to answer the questions if d is the depth of the water in centimeters and t is the time in seconds (see figure).

(a) Explain why d is a function of t.

(b) Determine the domain and range of the function.

(c) Sketch a possible graph of the function.



85. ***Modeling Data*** The table shows the average numbers of acres per farm in the United States for selected years. *(Source: U.S. Department of Agriculture)*

Year	1950	1960	1970	1980	1990	2000
Acreage	213	297	374	426	460	434

(a) Plot the data where A is the acreage and t is the time in years, with $t = 0$ corresponding to 1950. Sketch a freehand curve that approximates the data.

(b) Use the curve in part (a) to approximate $A(15)$.

86. ***Automobile Aerodynamics*** The horsepower H required to overcome wind drag on a certain automobile is approximated by

$$H(x) = 0.002x^2 + 0.005x - 0.029, \qquad 10 \le x \le 100$$

where x is the speed of the car in miles per hour.

(a) Use a graphing utility to graph H.

(b) Rewrite the power function so that x represents the speed in kilometers per hour. [Find $H(x/1.6)$.]

87. ***Think About It*** Write the function

$$f(x) = |x| + |x - 2|$$

without using absolute value signs. (For a review of absolute value, see Appendix D.)

88. ***Writing*** Use a graphing utility to graph the polynomial functions $p_1(x) = x^3 - x + 1$ and $p_2(x) = x^3 - x$. How many zeros does each function have? Is there a cubic polynomial that has no zeros? Explain.

89. Prove that the function is odd.

$$f(x) = a_{2n+1}x^{2n+1} + \cdot \cdot \cdot + a_3x^3 + a_1x$$

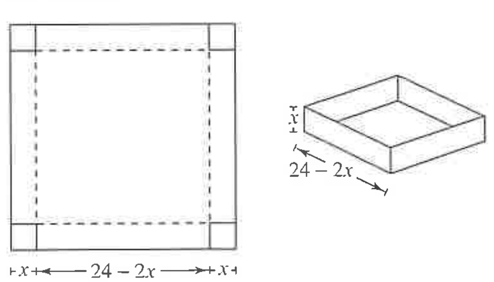
90. Prove that the function is even.

$$f(x) = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdot \cdot \cdot + a_2x^2 + a_0$$

91. Prove that the product of two even (or two odd) functions is even.

92. Prove that the product of an odd function and an even function is odd.

93. ***Volume*** An open box of maximum volume is to be made from a square piece of material 24 centimeters on a side by cutting equal squares from the corners and turning up the sides (see figure).



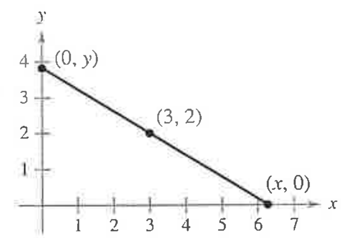
(a) Write the volume V as a function of x, the length of the corner squares. What is the domain of the function?

(b) Use a graphing utility to graph the volume function and approximate the dimensions of the box that yield a maximum volume.

(c) Use the *table* feature of a graphing utility to verify your answer in part (b). (The first two rows of the table are shown.)

Height, x	**Length and Width**	**Volume, V**
1	$24 - 2(1)$	$1[24 - 2(1)]^2 = 484$
2	$24 - 2(2)$	$2[24 - 2(2)]^2 = 800$

94. ***Length*** A right triangle is formed in the first quadrant by the x- and y-axes and a line through the point $(3, 2)$ (see figure). Write the length L of the hypotenuse as a function of x.



True or False? **In Exercises 95–98, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

95. If $f(a) = f(b)$, then $a = b$.

96. A vertical line can intersect the graph of a function at most once.

97. If $f(x) = f(-x)$ for all x in the domain of f, then the graph of f is symmetric with respect to the y-axis.

98. If f is a function, then $f(ax) = af(x)$.

Putnam Exam Challenge

99. Let R be the region consisting of the points (x, y) of the Cartesian plane satisfying both $|x| - |y| \le 1$ and $|y| \le 1$. Sketch the region R and find its area.

100. Consider a polynomial $f(x)$ with real coefficients having the property $f(g(x)) = g(f(x))$ for every polynomial $g(x)$ with real coefficients. Determine and prove the nature of $f(x)$.

These problems were composed by the Committee on the Putnam Prize Competition.
© The Mathematical Association of America. All rights reserved.

Section P.5 "Solving Equations and Inequalities"

EQ: "How do you solve equations and inequalities algebraically and graphically?"

A. Solve for "x" algebraically: [Hint: Try factoring, quadratic formula, and for absolute value you should always have 2 cases]

1). $4x^3 - 16x^2 - 3x + 12 = 0$

2). $x^2 - 4x = -1$

3). $\left|\frac{x}{2} - 1\right| = 1$

4). $|2x - 5| < 4$

5). $|3x + 2| \geq 3$

6). $3x^3 + x^2 \leq 14x$

B. Solve for "x" graphically: [Hint: First set equal to zero]

7). $2x^3 + 7x^2 = 2$

8). $\left|5 - \frac{x}{2}\right| \leq 1$

9). $2x^3 - x^5 > 2 - 5x^2$

P. 6 " Inverse Functions"

EQ: "How do you find the inverse function?"

A. Notes

1). Define the term "One-to-one Function"

2). If f(x) is a one-to-one function, then $f^{-1}(x)$ [the inverse of f(x)] is also

a one-to-one function .

- How do you create an inverse function?

Algebraically?

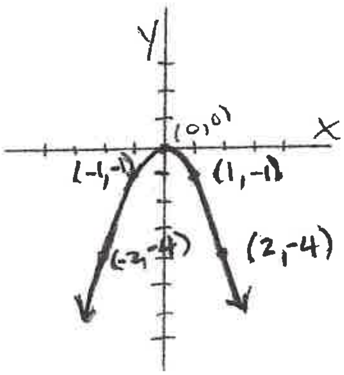
- Switch x & y
- Solve for y
- Check domain

Graphically?

* Graph the points (y,x)
[switch them!]

B. Work on the following problems:

3). Using the parabola given, graph the inverse relation



4). Find the inverse function to $y = 2 - x^3$

5). Find the inverse function to $y = \sqrt{x-4}$

6). Find the inverse function to $y = (x - 1)^2$, if $x \leq 1$

7). Find the inverse function to $y = \frac{x}{x-2}$

Section P.7 "Review of Trigonometry"

EQ: " How do you evaluate & graph trig functions? How do you solve trig equations?"

A. Radians vs. Degrees (See Unit Circle on the last page)

1). Change 20° to Radians [Note: Use $\pi = 180°$]

2). Change $\frac{-3\pi}{4}$ to Degrees

B. Find the following trig values

[Hint: Use the unit circle and/or the two special triangles - 30°-60°-90° and 45°-45°-90°]

3). $\sin(\pi/4)$

4). $\cos(3\pi/2)$

5). $\tan(5\pi/6)$

6). $\sec(-\pi/3)$

C. Graphs of Trig Functions

7). What is the period of each trig function?

8). For Sine and Cosine, how do you find the amplitude and centerline?

9). Graph: $y = 3\cos(x) - 1$

10). Graph: $y = -2\sin(3x)$

D. Solving Equations & Inequalities Algebraically

→ **Solve each equation for "x" over the interval [0, 2π]:**

11). $\sin(x) = 0$

12). $\cot(x) = -1$

13). $\csc(x) = -\frac{2}{\sqrt{3}}$

14). $2\sin^2(x) - \sin(x) = 0$

15). $\cos(x) + 1 = 2\sin^2(x)$

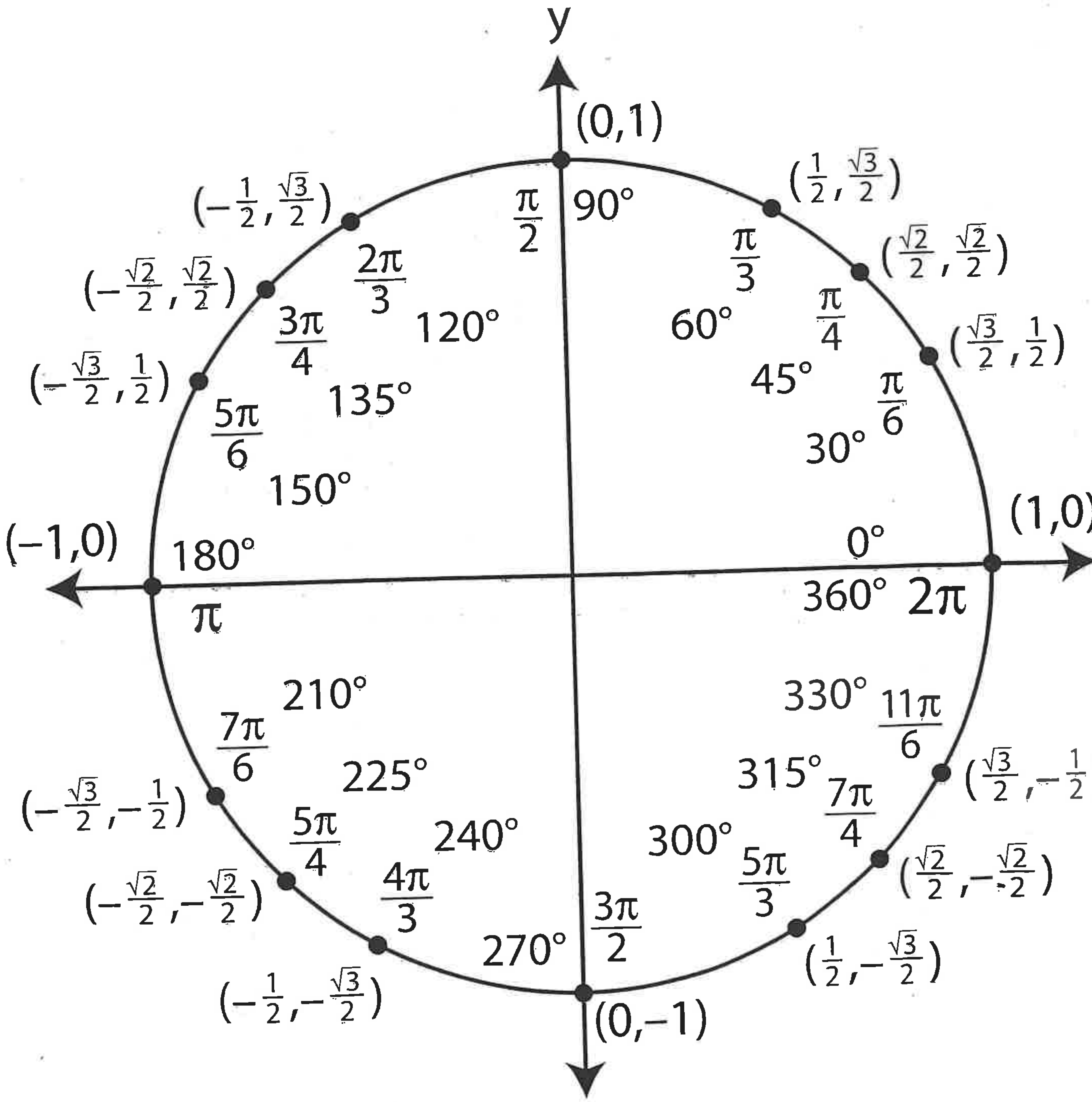
16). $2\sin^2(x) = 1$

E. Solving Equations & Inequalities Graphically:

17). Solve for "x" on $[0,2\pi]$: $\sin(x) = \cos(x)$

18). Solve for "x" : $\cos(x) - 2x = 0$

19) . Solve for "x" : $\cos(x) \geq x$



Note: $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

EAI EDUCATION ©2007 EAI Education, Oakland, NJ 07436 U.S.A. • www.EAIeducation.com • All Rights Reserved ® denotes US Pat. & TM Office.